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The moving plate thermometer

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Abstract

The adiabatic temperature displayed by the moving counterpart of Pohlhausen's classical *plate thermometer* is investigated as a function of the Prandtl number Pr. While in the classical case the heat release by viscous dissipation is due to the Blasius flow (of free stream velocity U_0), in the present case it is due to the boundary-layer flow induced by a continuous plane surface moving with the uniform velocity U_0 in a quiescent fluid (Sakiadis flow). It is found that the dimensionless adiabatic surface temperature ($=$ *recovery factor*) in both cases is given by each a monotonically increasing function of Pr. The two functions take the same value 1 at $Pr = 1$, but below and above of $Pr = 1$, they deviate from each other significantly. The recovery factor of the *moving plate thermometer* is investigated analytically and numerically by using the series solution of the problem obtained by the Merkin transformation method.

Keywords: Adiabatic temperature; Plate thermometer; Recovery factor; Wall driven flows; Merkin transformation

1. Introduction

The aim of the present paper is to compare the effect of viscous self heating of a fluid in two classical boundary layer flows. The one is the *Blasius flow* driven by a free stream of velocity U_0 over a semi-infinite flat plate. The other one is a *Sakiadis flow*, namely the boundary layer flow induced by a continuous plane surface which issues from a narrow linear slot and moves with constant velocity U_0 in a quiescent fluid (Schlichting and Gersten, [1], pp. 156 and 177, respectively, as well as Sakiadis [2,3]). Assuming that the surface in both cases is adiabatic (i.e. impermeable to heat), its temperature T_{ad} increases due to the volumetric heat generation by viscous dissipation, exceeding the ambient temperature T_∞ of the fluid by a constant quantity ΔT, $T_{ad} = T_\infty + \Delta T$. In case of the Blasius flow, the adiabatic temperature increase ΔT of the plate was calculated for moderate values of the Prandtl number Pr long time ago by Ernst Pohlhausen and reported in his seminal ZAMM-paper [4]. This device, i.e. the adiabatic plate in a uniform stream, was named by Pohlhausen *Plattenthermometer* (plate thermometer). Later on, the temperature displayed by Pohlhausen's plate thermometer was calculated by Eckert and Drewitz [5] for several other values of Pr up to $Pr = 1000$. More accurate results for large Pr were reported by Meksyn [6]. Simple scaling relationships between ΔT and Pr have been obtained by Gersten and Körner [7] for $Pr \ll 1$, and by Narashima and Vasantha [8] for $Pr \gg 1$. Various correlating equations for ΔT and Pr were proposed by Churchill and Char [9].

The goal of the present paper is to examine the Pr-dependence of the adiabatic temperature increase ΔT displayed by the *moving* counterpart of Pohlhausen's plate thermometer, which will be referred to as *moving plate thermometer*. As we are aware, this problem has not been investigated until now. Our approach is based on a series solution of the problem, obtained by the Merkin transformation method [10].

2. Basic equations

Both the Pohlhausen plate thermometer and its moving counterpart are governed by the same continuity, momentum and energy equations, which in the boundary layer approximation read [1]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2}$$

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Nomenclature

A	dimensionless expansion coefficients, Eq. (21)
c_p	specific heat capacity at constant pressure, Eq. (1)
f	dimensionless stream function, Eq. (4)
Pr	Prandtl number, $Pr = \upsilon/\alpha$
r	recovery factor, Eq. (11)
T	temperature
T_0	$= U_0^2/(2c_p)$ stagnation point temperature
U_0	free stream and surface velocity, Eqs. (2), (3)
u	dimensional longitudinal velocity, Eq. (4)
v	dimensional transversal velocity, Eq. (4)
z	dimensionless independent variable, Eq. (17)
x	dimensional wall coordinate
y	dimensional transversal coordinate
Y	dimensionless dependent variable, Eq. (17)

Greek symbols

α	thermal diffusivity, Eq. (1)
δ	thickness temperature boundary layer
ε	tuning parameter, Eq. (30)
υ	kinematic viscosity, Eq. (1)
η	similarity independent variable, Eq. (4)
θ	dimensionless temperature, Eq. (5)

Subscripts

∞	conditions at infinity
M	Moving plate thermometer
P	Pohlhausen's plate thermometer

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\upsilon}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{1}$$

(The coordinate system corresponds respectively to Figs. 6.5 and 7.6 of [1].)

The boundary conditions for the Pohlhausen plate thermometer are

$$\begin{aligned} &u(x,0)=0, \qquad v(x,0)=0, \qquad u(x,\infty)=U_0 \\ &\frac{\partial T}{\partial y}(x,0)=0, \qquad T(x,\infty)=T_\infty \quad (\text{“Pohlhausen”}) \end{aligned} \tag{2}$$

The same boundary conditions hold also for the moving plate thermometer, except for the conditions on the parallel velocity component which are

$$u(x,0)=U_0, \qquad u(x,\infty)=0 \quad (\text{“Moving”}) \tag{3}$$

The solution for the velocity field can be obtained in both cases in the usual way [1] from the same stream function

$$\psi(x,y)=\sqrt{2\upsilon x U_0}\,f(\eta), \qquad \eta=\sqrt{\frac{U_0}{2\upsilon x}}\cdot y \tag{4}$$

The corresponding temperature field has the form

$$T(x,y)=T_\infty + T_0\cdot\theta(\eta), \qquad T_0=\frac{U_0^2}{2c_p} \tag{5}$$

In the above equations, $f=f(\eta)$ is the similar stream function, $\theta=\theta(\eta)$ the similar temperature field and $T_0=U_0^2/(2c_p)$ denotes the stagnation point temperature. The functions $f=f(\eta)$ and $\theta=\theta(\eta)$ satisfy the equations, [1–3],

$$f''' + ff'' = 0 \tag{6}$$

$$\theta'' + Prf\theta' = -2Prf''^2 \tag{7}$$

for both plate thermometers, but the (velocity) boundary conditions are different, namely

$$\begin{aligned} &f(0)=0, \qquad f'(0)=0, \qquad f'(\infty)=1 \\ &\theta'(0)=0, \qquad \theta(\infty)=0 \quad (\text{“Pohlhausen”}) \end{aligned} \tag{8}$$

for the Pohlhausen thermometer, and

$$\begin{aligned} &f(0)=0, \qquad f'(0)=1, \qquad f'(\infty)=0 \\ &\theta'(0)=0, \qquad \theta(\infty)=0 \quad (\text{“Moving”}) \end{aligned} \tag{9}$$

for the moving plate thermometer, respectively (the primes denote differentiation with respect to η). The velocity boundary value problems (i.e. the f-problems) are decoupled from the temperature problems. Their numerical solutions are well known [1] and the corresponding values of the similar wall shear stress $f''(0)$ are $f''(0)=0.46960000$ (“Pohlhausen”) and $f''(0)=-0.62755488$ (“Moving”), respectively.

According to Eq. (5), the *adiabatic temperature* displayed by the plate thermometers is

$$T_{\mathrm{ad}} \equiv T|_{y=0} = T_\infty + \Delta T, \qquad \Delta T = T_0\theta(0) \tag{10}$$

where the dimensionless wall temperature $\theta(0)$ coincides with the *recovery factor*,

$$\theta(0)=\frac{T_{\mathrm{ad}}-T_\infty}{T_0} \equiv r(Pr) \tag{11}$$

Consequently, the dependence of the recovery factor r on the Prandtl number is the feature of main physical and engineering interest of the problem.

3. The Pohlhausen solution

As shown already by Pohlhausen [4], the solution of Eq. (7) which satisfies the boundary conditions (8) can be expressed in terms of the similar shear stress distribution $f''(\eta)$ in a double integral form, which according to [1] can be transcribed to

$$\theta(\eta)=2Pr\int_\eta^\infty\left[f''(\xi)\right]^{Pr}\left(\int_0^\xi\left[f''(z)\right]^{2-Pr}\mathrm{d}z\right)\mathrm{d}\xi \tag{12}$$

Thus, the recovery factor of Pohlhausen's plate thermometer is obtained as

$$r(Pr)=2Pr\int_0^\infty\left[f''(\xi)\right]^{Pr}\left(\int_0^\xi\left[f''(z)\right]^{2-Pr}\mathrm{d}z\right)\mathrm{d}\xi \tag{13}$$

For $Pr = 1$, the integrals can easily be evaluated and yield

$$\theta(\eta) = 1 - f'^2(\eta) \quad (Pr = 1) \tag{14}$$

and

$$r(1) = 1 \tag{15}$$

respectively. Since the Blasius equation (6) along with the f-boundary conditions (8) does not admit solution in a closed analytical form, the functions $f''(\eta)$ and $f'(\eta)$ have been evaluated numerically [1–8].

4. Solution for the moving plate thermometer

4.1. The double integral solution

The double integral solution (12), as well as the expression (13) of the recovery factor are valid also in the case of moving plate thermometer. However, due to the different velocity boundary conditions, the function $f(\eta)$ is different, and thus the coincidence is formal. This aspect becomes manifest already in the case $Pr = 1$, in which Eq. (12) for the moving plate thermometer yields

$$\theta(\eta) = \left[2 - f'(\eta)\right] \cdot f'(\eta) \quad (Pr = 1) \tag{16}$$

Although Eqs. (14) and (16) give for $Pr = 1$ the same recovery factor $r(1) = 1$, the temperature fields (14) and (16) deviate from each other almost everywhere (details in Section 5 below).

4.2. Series solution by the Merkin transformation method

To calculate the recovery factor of the moving plate thermometer according to Eq. (13), the standard method would be to solve the corresponding boundary value problem for $f''(\eta)$ numerically. In the present section, however, we follow an alternate approach and give an analytical series solution for $f''(\eta)$ with the aid of the Merkin transformation method, [10].

The basic feature of the Merkin transformation is that it reverses role of the stream function f in the boundary value problem (6), (9) from that of the old dependent variable to that of a new independent variable $\phi \equiv f_\infty - f$ and at the same time, it transfers the role of the dependent variable from f to $p(\phi) \equiv \mathrm{d}f/\mathrm{d}\eta$, where f_∞ denotes the similar entrainment velocity, $f_\infty = f(\infty)$. For later convenience, we modify the Merkin transformation slightly by changing instead of ϕ and $p(\phi)$ to a new independent variable z and to a new dependent one, $Y = Y(z)$, which we define as follows [11]:

$$z = \frac{\phi}{f_\infty} = 1 - \frac{f}{f_\infty}, \qquad Y = \frac{p(\phi)}{f_\infty^2} = \frac{1}{f_\infty^2}\frac{\mathrm{d}f}{\mathrm{d}\eta} \tag{17}$$

Thus, the third-order boundary value problem (6), (9) reduces to the second-order one

$$\frac{\mathrm{d}}{\mathrm{d}z}\left(Y\frac{\mathrm{d}Y}{\mathrm{d}z}\right) + (z-1)\frac{\mathrm{d}Y}{\mathrm{d}z} = 0 \tag{18}$$

$$Y(0) = 0, \qquad Y(1) = \frac{1}{f_\infty^2} \tag{19}$$

where the first condition (19) has been obtained from $f'(\infty) = 0$, and the second one from $f(0) = 0$ and $f'(0) = 1$. Furthermore, the expression (13) of the recovery factor $r(Pr) = \theta(0)$ transcribes with the new variables z and $Y = Y(z)$ to

$$r(Pr) = 2Pr f_\infty^4 \int_0^1 \left[Y(\xi)\right]^{Pr-1}\left[\frac{\mathrm{d}Y(\xi)}{\mathrm{d}\xi}\right]^{Pr} \times \left(\int_\xi^1 \left[Y(z)\right]^{1-Pr}\left[\frac{\mathrm{d}Y(z)}{\mathrm{d}z}\right]^{2-Pr}\mathrm{d}z\right)\mathrm{d}\xi \tag{20}$$

This expression involves only the new independent variable Y and its first derivative $\mathrm{d}Y/\mathrm{d}z$, instead of the second derivatives $f''(\eta)$ of the similar stream function $f(\eta)$ present in Eq. (13).

Now, looking for the solution of the boundary value problem (18), (19) in the power series form [10]

$$Y = \sum_{n=0}^{\infty} A_n z^n \tag{21}$$

one obtains for the coefficients A_n the system of equations (see also [11]),

$$\sum_{n=0}^{k}(n+1)\left[(n+2)A_{n+2}A_{k-n} + (k-n+1)A_{n+1}A_{k-n+1}\right] = (k+1)A_{k+1} - kA_k, \quad k = 0, 1, 2, \ldots \tag{22}$$

The boundary condition $Y(0) = 0$ implies $A_0 = 0$. Thus, one obtains from Eq. (22) for the next two coefficients the values

$$A_1 = 1, \quad A_2 = -1/4 \tag{23}$$

The subsequent coefficients $A_3, A_4, A_5, \ldots$ can then be obtained recursively according to

$$A_k = \frac{1-k}{k^2}A_{k-1} - \frac{1+k}{2k}\cdot\sum_{n=2}^{k-1}A_n A_{k-n+1}, \quad k = 3, 4, 5, \ldots \tag{24}$$

Specifically,

$$A_3 = \frac{1}{72}, \quad A_4 = \frac{1}{576}, \quad A_5 = \frac{11}{86400}$$
$$A_6 = -\frac{1}{115200}, \quad \ldots \tag{25}$$

Thus, the second boundary condition (19) yields for f_∞ the explicit equation

$$f_\infty = \left(\sum_{k=1}^{\infty} A_k\right)^{-1/2} \tag{26}$$

the wall shear stress $f''(0)$ is obtained as

$$f''(0) = -f_\infty \frac{\mathrm{d}Y}{\mathrm{d}z}\bigg|_{z=1} = -\frac{\sum_{k=1}^{\infty} kA_k}{\left(\sum_{k=1}^{\infty} A_k\right)^{1/2}} \tag{27}$$

and the recovery factor (20) becomes

$$\frac{r(Pr)}{2Pr} = \frac{\int_0^1 \left(\sum_{k=1}^{\infty} A_k \xi^k\right)^{Pr-1} \left(\sum_{k=1}^{\infty} k A_k \xi^{k-1}\right)^{Pr}}{\left(\sum_{k=1}^{\infty} A_k \xi^k\right)^4} \times \left[\int_{\xi}^{1} \left(\sum_{k=1}^{\infty} A_k z^k\right)^{1-Pr} \left(\sum_{k=1}^{\infty} k A_k z^{k-1}\right)^{2-Pr} \mathrm{d}z\right] \mathrm{d}\xi \tag{28}$$

Eqs. (25) show that with increasing n the coefficients of the series (21) decrease rapidly. Already with the first 10 term of the series involved, one obtains from Eqs. (26) and (27) the values of f_∞ and $f''(0)$ with an accuracy of 7 digits,

$$f_\infty = 1.142773, \qquad f''(0) = -0.627554 \tag{29}$$

We mention that the rate of convergence of the series occurring in Eqs. (21) and (26)–(28) can be accelerated once more with the aid of the classical Euler–Knopp type series transformation, [12], or with its improved form proposed by Gabutti and Lyness [13]. Applied, e.g., to the series (21), the transformation of Gabutti and Lyness gives

$$Y(z) = \sum_{n=0}^{\infty} \frac{n!}{(1-\varepsilon)^{n+1}} \left(\sum_{j=0}^{n} \frac{(-\varepsilon)^{n-j}}{(n-j)!\,j!} A_j z^j\right) \tag{30}$$

Here ε is a *tuning parameter* which can be chosen at convenience (for $\varepsilon = -1$ one recovers the classical Euler–Knopp type transformation).

5. Discussion

For a clear distinction between the quantities which refer to the Pohlhausen and the moving plate thermometer, hereafter the subscripts P (Pohlhausen) and M (Moving) will be used.

As already mentioned, for $Pr = 1$ the recovery factors $r(Pr)$ of the two thermometers coincide, $r_P(1) = r_M(1) = 1$. In spite of this fact, the two temperature fields (14) and (16) deviate from each other almost everywhere. This latter feature is illustrated in Fig. 1. The 1%-thicknesses of these two similar temperature boundary layers, i.e. the values $\eta = \delta_P$ and $\eta = \delta_M$ which solve the respective equations $\theta(\delta) = \theta(0)/100$, also differ from each other, namely, $\delta_P = 3.72608$ and $\delta_M = 5.10821$.

For $Pr \neq 1$, on the other hand, the difference between the two temperature fields $\theta_P(\eta)$ and $\theta_M(\eta)$, the recovery factors $r_P(Pr)$ and $r_M(Pr)$, as well as the 1%-thicknesses $\delta_P(Pr)$ and $\delta_M(Pr)$, may become quite large, as shown in Figs. 2 and 3. The plots of the 1%-thicknesses $\delta_P(Pr)$ and $\delta_M(Pr)$ in the Prandtl number range $0.1 \leqslant Pr \leqslant 7$ are shown in Fig. 4. While for $Pr \gg 1$, δ_P and δ_M approach each other (both going to zero as $Pr \to \infty$), for small values of Pr, δ_M becomes much larger than δ_P. For instance, for $Pr = 10^{-3}$, one has $\delta_M(10^{-3}) = 4031.00$, while $\delta_P(10^{-3}) = 82.6067$. Furthermore, in Fig. 5 the recovery factors $r(Pr)$ of the two thermometers as functions of the Prandtl number are compared to each other in the range $0 < Pr \leqslant 7$. It is seen once more that $r_P(1) = r_M(1) = 1$, while $r_P(Pr) < r_M(Pr)$ for $Pr < 1$ and $r_P(Pr) > r_M(Pr)$ for $Pr > 1$. Therefore, the behavior of the recovery factors of the two plate thermometers is basically different both for small and large Prandtl numbers. In the range $Pr \ll 1$, where according to Gersten and Körner [7] $r_P(Pr) \approx 0.9254 \cdot Pr^{1/2}$, the recovery factor $r_M(Pr)$ of the moving plate thermometer increases with increasing values of Pr much steeper. Already at $Pr = 10^{-3}$, it reaches the value $r_M(10^{-3}) = 0.713753$, while the exact value of $r_P(Pr)$ at this Prandtl number is only $r_P(10^{-3}) = 0.029458$, i.e., more than 24 times smaller than r_M. At large values of Pr, where according to Narashima and Vasantha [8] $r_P(Pr) = 1.922 \cdot (Pr + 0.805)^{1/3} - 1.341$, the situation becomes reversed. For $Pr = 10^3$, e.g., one has $r_M(10^3) = 3.39338$ and $r_P(10^3) = 17.892$, i.e., in this case r_P is more than 5 times larger than r_M.

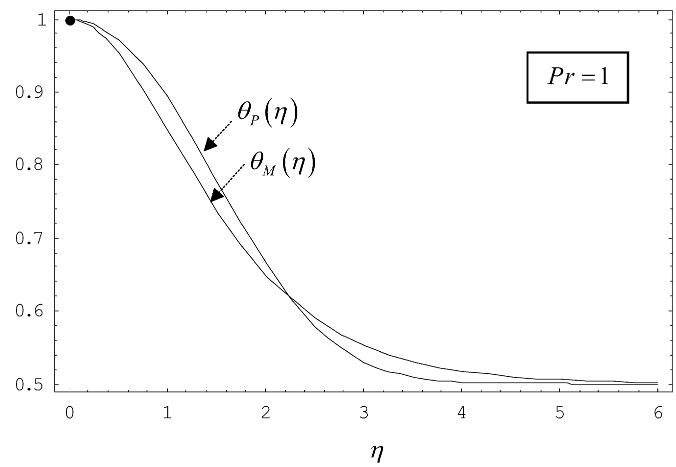

Fig. 1. The dimensionless adiabatic temperature profiles $\theta_P(\eta)$ and $\theta_M(\eta)$ of the Pohlhausen and the moving plate thermometer, respectively, corresponding to $Pr = 1$. In this (and only in this) case the two recovery factors (marked by dots) are equal, $r_P = \theta_P(0) = r_M = \theta_M(0) = 1$. The 1%-thicknesses $\delta_P(Pr)$ and $\delta_M(Pr)$ of the two temperature boundary layers are $\delta_P(1) = 3.72608$ and $\delta_M(1) = 5.10821$.

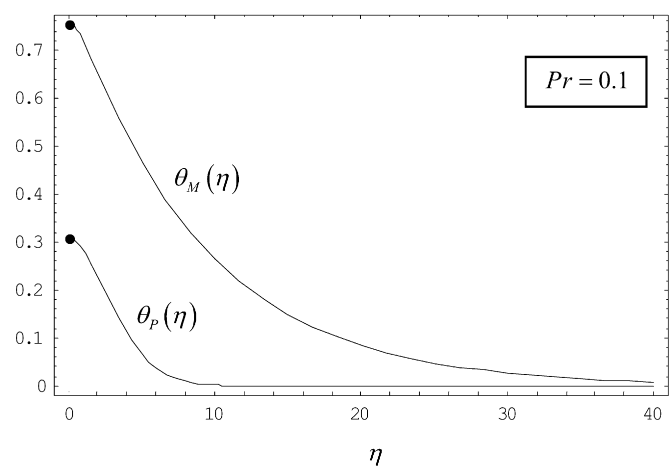

Fig. 2. The dimensionless adiabatic temperature profiles $\theta_P(\eta)$ and $\theta_M(\eta)$ of the Pohlhausen and the moving plate thermometer, respectively, corresponding to $Pr = 0.1$. The two recovery factors (marked by dots) are, $r_P = 0.307308$ and $r_M = 0.755620$, i.e., $r_M = 2.46 r_P$. The 1%-thicknesses $\delta_P(Pr)$ and $\delta_M(Pr)$ of the two temperature boundary layers are $\delta_P(0.1) = 0.28956$ and $\delta_M(0.1) = 41.0626$.

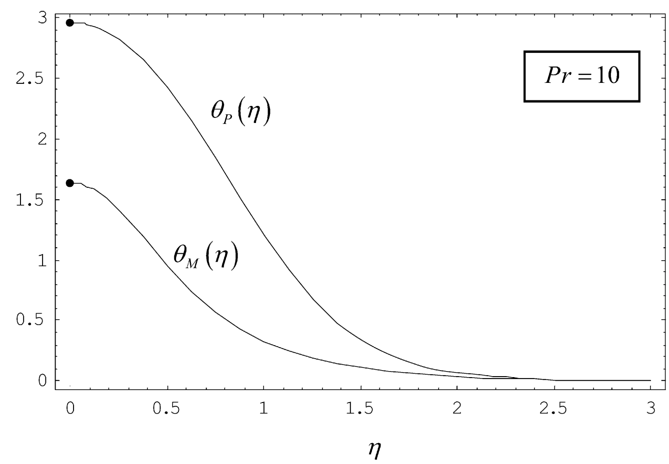

Fig. 3. The dimensionless adiabatic temperature profiles $\theta_P(\eta)$ and $\theta_M(\eta)$ of the Pohlhausen and the moving plate thermometer, respectively, corresponding to $Pr = 10$. The two recovery factors (marked by dots) are, $r_P = 2.961587$ and $r_M = 1.638258$, i.e., $r_M = 0.55 r_P$. The 1%-thicknesses $\delta_P(Pr)$ and $\delta_M(Pr)$ of the two temperature boundary layers are $\delta_P(10) = 2.2464$ and $\delta_M(10) = 2.35127$.

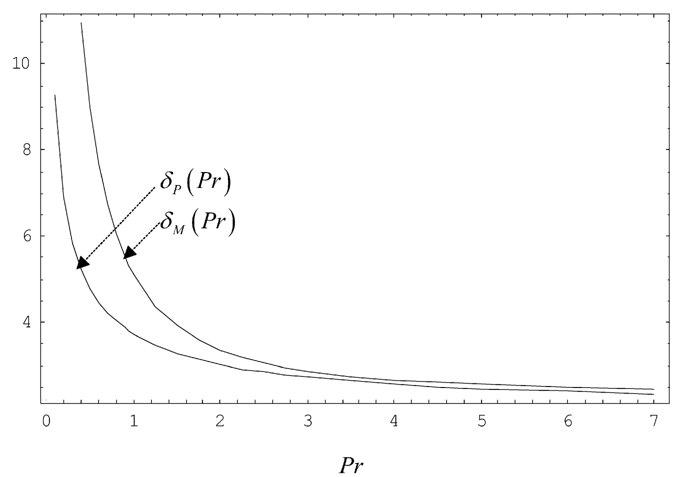

Fig. 4. Plots of the 1%-thicknesses $\delta_P(Pr)$ and $\delta_M(Pr)$ of the two temperature boundary layers in the Prandtl number range $0.1 \leqslant Pr \leqslant 7$.

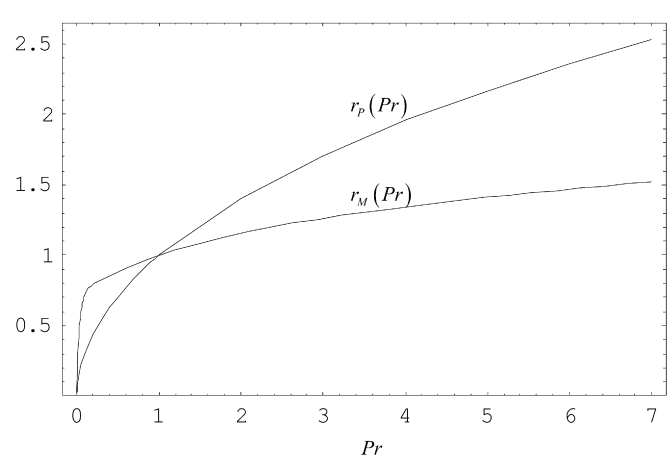

Fig. 5. Plots of the recovery factors $r_P(Pr)$ and $r_M(Pr)$ of the Pohlhausen and the moving plate thermometer, respectively, as functions of the Prandtl number Pr. For $Pr = 1$ the two recovery factors coincide, $r_P(1) = r_M(1) = 1$, but for $Pr \ll 1$ and $Pr \gg 1$, they deviate from each other substantially.

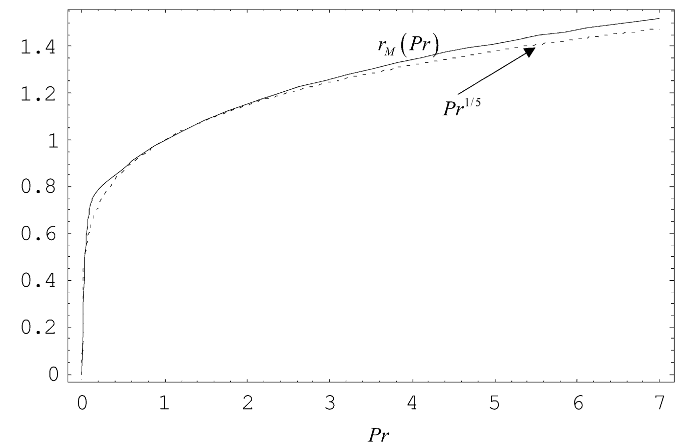

Fig. 6. Exact values of the recovery factor $r_M(Pr)$ compared to the approximate ones (dashed curve) given by the correlating equation (37), $r_M(Pr) \approx Pr^{1/5}$.

Table 1
Comparison of the exact values of the recovery factor of the moving plate thermometer for air and water to the values obtained from the correlating equation (31), $r_M(Pr) \approx Pr^{1/5}$. The values of Pr are taken at 20 °C and atmospheric pressure, [14]

Fluid	Pr	$r_M(Pr)$		Deviation,
		Exact	$Pr^{1/5}$	Exact-$Pr^{1/5}$
Dry air	0.72	0.940873	0.936411	0.47%
Water	7.07	1.522487	1.478713	2.87%

A remarkable feature of the moving plate thermometer is that its recovery factor $r_M(Pr)$ scales with Pr according to the simple correlating equation

$$r_M(Pr) \approx Pr^{1/5} \tag{31}$$

over a large range of values of the Prandtl number. This scaling property is illustrated in Fig. 6. For $Pr = 1$, Eq. (31) is exact, while both below and above of $Pr = 1$ it yields values close to the exact values of $r_M(Pr)$ obtained either from Eq. (16) for $\eta = 0$, or from Eq. (28) (or from the direct numerical solution the boundary value problem (6), (7), (9)). The performance of the correlating equation compared to the exact values of $r_M(Pr)$, can be seen in Table 1 for the case of air and water.

6. Summary and conclusions

The adiabatic temperature increase ΔT displayed by the uniformly moving plate thermometer has been compared in the present paper to that of Pohlhausen's classical (resting) plate thermometer. The surface temperature increment ΔT above the ambient temperature, nondimensionalized with the aid of the stagnation point temperature T_0, specifies the recovery factor $r = r(Pr)$, which is the quantity of basic physical and engineering interest of the problem. The main results of the paper can be summarized as follows.

1. For $Pr = 1$ the recovery factors of the two thermometers coincide, $r_P(1) = r_M(1) = 1$. In spite of this fact, the two temperature fields $\theta_P(\eta)$ and $\theta_M(\eta)$ deviate from each other almost everywhere. The 1%-thicknesses $\delta_P =$

3.72608 and $\delta_M = 5.10821$ of these two temperature boundary layers differ from each other sensitively.

2. For all $Pr < 1$, the inequality $r_M(Pr) > r_P(Pr)$, and for all $Pr > 1$, the inequality $r_M(Pr) < r_P(Pr)$ holds.
3. For the moving plate thermometer, the correlating equation $r_M(Pr) = Pr^{1/5}$ is applicable in a large range of the Prandtl number Pr (e.g., from $Pr = 0.72$, air, to $Pr = 7.07$, water, it applies with an accuracy of 0.47% to 2.87%).
4. The 1%-thicknesses $\delta_P(Pr)$ and $\delta_M(Pr)$ of the two temperature boundary layers approach each other as $Pr \to \infty$, while for small values of Pr, δ_M becomes much larger than δ_P.

We may conclude that, although the boundary layer flow induced over a resting semi-infinite plate by a uniform free stream (Blasius flow), on the one hand, and the boundary layer flow induced by a uniformly moving continuous plane surface in a quiescent fluid (Sakiadis flow), on the other hand seem to be nearly identical flow phenomena, the viscous self heating effect is able to produce a significant physical distinction between these two flows.

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